2010
HSC TRIAL
EXAMINATION PAPER

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks – 120

- Attempt Questions 1–10
- All questions are of equal value

THIS PAPER CANNOT BE RELEASED IN PUBLIC UNTIL AFTER 27TH AUGUST 2010
This paper is used with the understanding that it has a Security Period.

Total Marks – 120
Attempt Questions 1–10
All questions are of equal value

Answer each question in a SEPARATE writing booklet.

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) Evaluate $\dfrac{2.12^4}{4.67 - 1.95}$ correct to three decimal places. **2**

(b) Simplify $\dfrac{2x + 1}{3} - \dfrac{x - 5}{5}$. **2**

(c) Solve $|2x - 3| < 5$ and graph the solution on the number line. **3**

(d) Find the exact value of **2**
$\tan\dfrac{2\pi}{3} - \sin\dfrac{\pi}{3}$.

(e) The gradient of a curve is given by $6x^2 - 4x + 1$. If the curve passes through the point (1,3), find the equation of the curve. **3**

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Question 2 (12 marks) Use a SEPARATE writing booklet.

(a) Differentiate with respect to x:

(i) $2x\log_e x$ **2**

(ii) $\sin^3 2x$ **2**

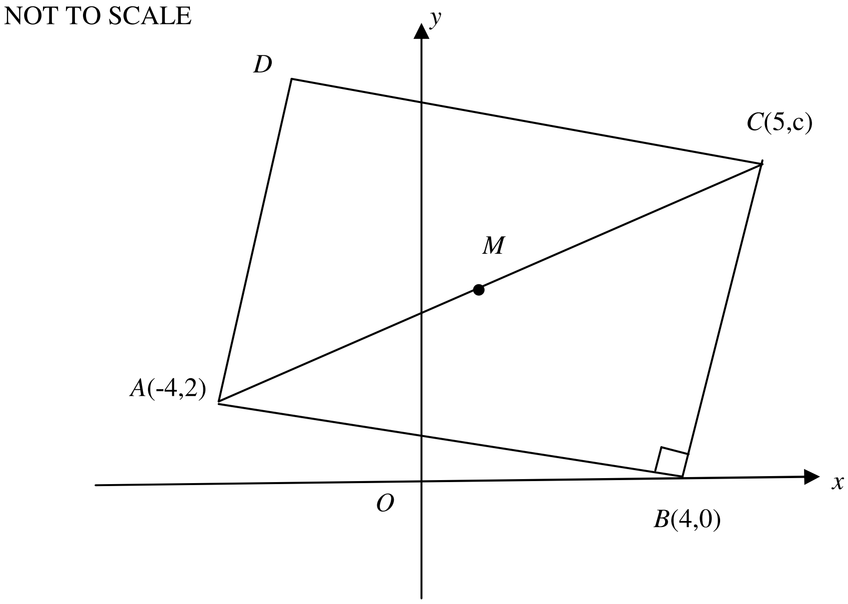
(b) (i) Find $\int e^{3x}\,dx$ **2**

(ii) Evaluate $\int_0^{\frac{\pi}{4}} \sin 2\theta \, d\theta$ **3**

(c) Find the equation of the tangent to the curve $y = e^{2x}$ at the point where $x = 1$. **3**

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) In the diagram, $ABCD$ is a rectangle. A, B and C are the points $(-4,2)$, $(4,0)$ and $(5,c)$ respectively.



(i) Find the gradient of AB. **1**

(ii) Show that the equation of BC is $4x - y - 16 = 0$. **2**

(iii) Show that $c = 4$. **1**

(iv) Find the mid-pint M of AC.
Hence, or otherwise, find the co-ordinates of D. **2**

(v) Find the area of the rectangle $ABCD$. **2**

(vi) Write down the area of ΔABM. **1**

(b) Consider the arithmetic series $7 + 11 + 15 + \cdots$. **3**
Find the first term in the series which exceeds 1000.

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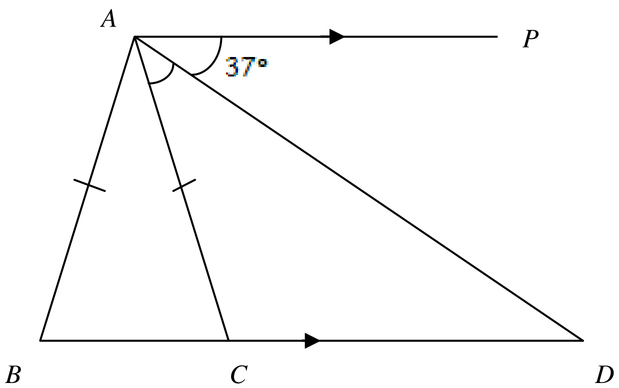
Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) In the diagram, ΔABC is an isosceles triangle with $AB = AC$.

AD bisects $\angle PAC$ and meets BC produced at D. $\dfrac{\dfrac{AP}{\square}}{BD}$. $\angle PAD = 37°$.



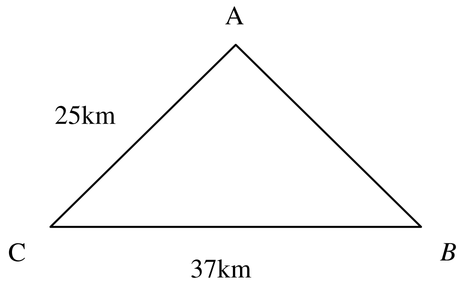
NOT TO SCALE

Copy or trace this diagram into your writing booklet.

(i) Find, giving reasons, $\angle ACD$. **2**

(ii) Hence find, giving reasons, $\angle BAC$. **2**

(b)



NOT TO SCALE

In the diagram, town *C* is due west of town *B*. The bearing of town *A* from *C* is 042°. *C* is 25km and 37km from *A* and *B* respectively.

Copy or trace this diagram into your writing booklet showing all the given information.

(i) Calculate the distance of *B* from *A*. **2**

(ii) Find the bearing of *A* from *B*. **2**

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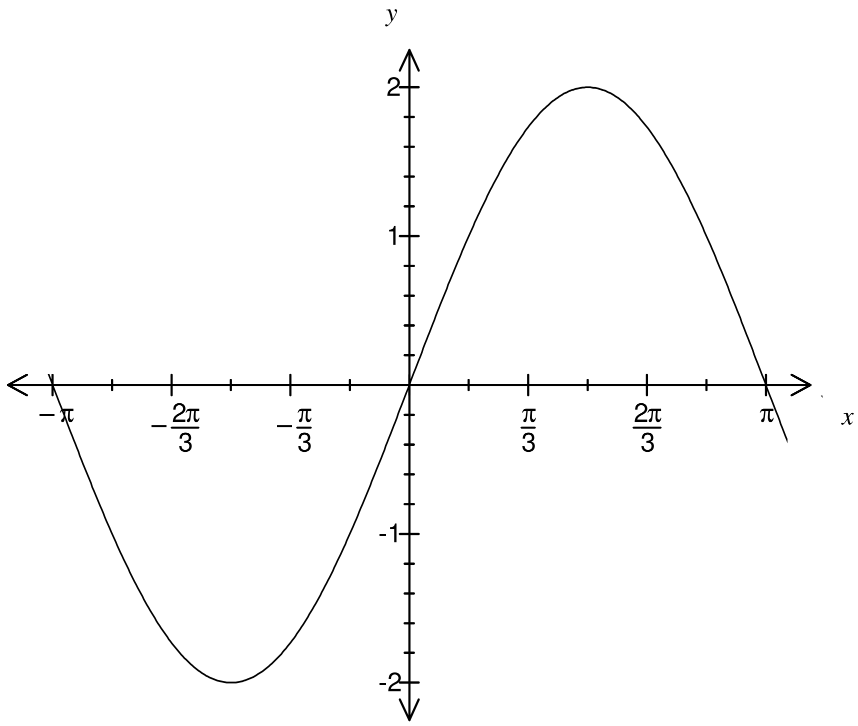
Question 4 continues on page 6

Question 4 (continued)

(c)



The diagram shows the graph of $y = 2\sin x$ for $-\pi \le x \le \pi$.
Copy or trace this diagram into your writing booklet.

(i) Draw the graph of $y = \frac{x}{3}$ in the same diagram. **1**

(ii) Find the exact area enclosed by the graphs $y = \frac{x}{3}$ and $y = 2\sin x$ between $x = -\frac{\pi}{3}$ and $x = \frac{\pi}{3}$. **3**

End of Question 4

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Question 5 (12 marks) Use a SEPARATE writing booklet.

(a) Consider the function $f(x) = x^3 - 3x^2$.

(i) Find the coordinates of the stationary points of the graph of $y = f(x)$ and determine their nature. **3**

(ii) Find the coordinates of the point of inflexion of the graph of $y = f(x)$. **2**

(iii) Find the values of x for which the graph of $y = f(x)$ is concave downwards. **1**

(iv) Sketch the graph of $y = f(x)$, showing the turning points, the point of inflexion and the points where the curve meets the x-axis. **2**

(v) Find the minimum value of the function $f(x)$ for $-2 \le x \le 3$. **1**

(b) The table below shows the velocity $v(t)$, in ms^{-1}, of a particle moving in a straight line at time t seconds.

t	0	1	2	3	4
$v(t)$	0	2	5	4	2

(i) Explain why the displacement x, in metres, of the particle for the time interval $0 \le t \le 4$ is given by the definite integral **1**

$$\int_0^4 v(t)dt.$$

(ii) Use Simpson's Rule with 5 function values to find an approximation for **2**

$$\int_0^4 v(t)dt.$$

Question 6 (12 marks) Use a SEPARATE writing booklet.

(a) The population of a mining town has been increasing over the past few years due to the increasing export of resources. Four years ago the population was estimated to be 2500 and today it is estimated at 4500. Assume the population P of the town grows according to the equation $P = P_0 e^{kt}$ where P_0 and k are constants and t is the time in years from the first estimate four years ago.

(i) Find the exact value of k. **3**

(ii) If the population continues to grow at this rate,

(α) estimate the population five years from now; **2**

(β) find when will the population in the town first exceed 7000. **1**

(b) A shop holds a lucky draw for its customers. Each customer draws two chips from a box containing a large number of chips and claims a prize according to the colours of the chips. 10% of the chips are red, 30% are blue and 60% are yellow. John draws two chips from the box. Find the probability of getting

(i) 2 red chips; **1**

(ii) at least one red chip; **2**

(iii) 2 chips of different colours. **3**

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Question 7 (12 marks) Use a SEPARATE writing booklet.

(a) (i) Show that the curve $y = \sqrt{x}$ and the line $y = \frac{x}{2}$ intersect at the points where $x = 0$ and 4. **1**

(ii) The area between the curve $y = \sqrt{x}$ and the line $y = \frac{x}{2}$ is rotated $360°$ about the x-axis. Find the volume of the solid of revolution. **3**

(b) Consider the quadratic function $f(x) = kx^2 + kx + (k - 3)$ where k is a constant.

(i) Find the discriminant of $f(x)$. Simplify your answer. **1**

(ii) If $f(x)$ is positive definite, find the range of values of k. **3**

("c) Consider the geometric series " $1 + \sin^{2}\theta + \sin^{4}\theta + \cdots$, where $0 <$

(i) Explain why the series has a limiting sum. **1**

(ii) Find the limiting sum of the series in terms of θ. **2**

(iii) Find the range of the values of the limiting sum. **1**

Question 8 (12 marks) Use a SEPARATE writing booklet.

(a) Solve the equation $\log_3(x-2)+\log_3 x = 1$. **3**

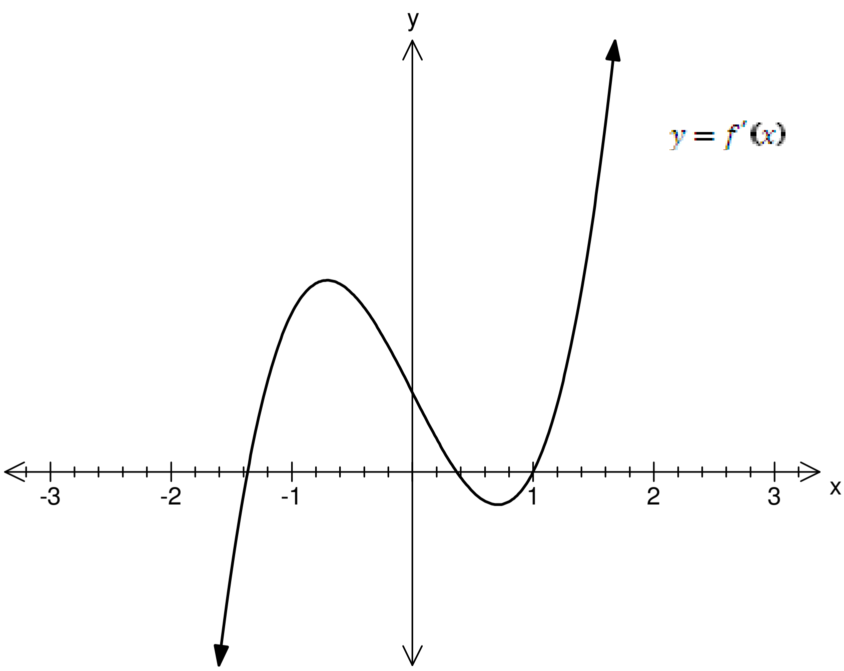
(b) A particle moves in a straight line so that its displacement x metres from the origin O at time t seconds is given by $x = t^3 - 3t$.

(i) When is the particle at rest? **2**

(ii) In what direction is the particle moving when $t = 1$. Give brief reasons to support your answer. **2**

(iii) Find the distance travelled in the first 3 seconds. **2**

(c) The graph of $y = f'(x)$ is shown below.



Draw the graph of $y = f(x)$, showing all the important features, given that it passes through the point (0,0). **3**

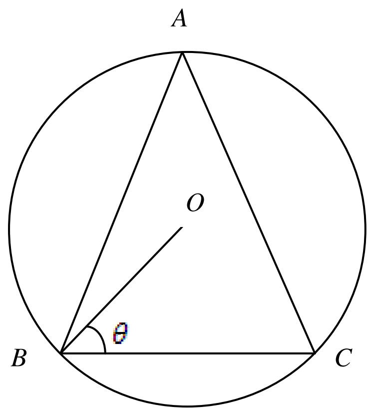
Question 9 (12 Marks) Use a SEPARATE writing booklet.

(a) (i) Solve $\sin x = \cos x$ for $0 \leq x \leq 2\pi$. **2**

(ii) On the same set of axes, sketch the graphs of $y = \sin x$ and $y = \cos x$ for $0 \leq x \leq 2\pi$. **1**

(iii) Hence write down the solution for $\sin x > \cos x$ for $0 \leq x \leq 2\pi$. **1**

(b)



NOT TO SCALE

In the diagram, O is the centre of the circle with fixed radius r cm. An isosceles triangle ABC is inscribed in the circle with $AB = AC$. $\angle OBC = \theta$.

(i) Find the length of the base BC in terms of r and θ. **1**

(ii) Show that the area S of ΔABC is given by **1**

$S = r^2 \cos\theta(1 + \sin\theta)$.

(iii) Show that $\dfrac{dS}{d\theta} = r^2(-2\sin^2\theta - \sin[\theta + 1).]$ **2**

(iv) Find the maximum area of ΔABC in terms of r. **4**
What type of triangle is ΔABC when its area is a maximum? Give brief reasons.

Question 10 (12 Marks) Use a SEPARATE writing booklet.

(a) The function $f(x)$ is defined by

$$f(x) = \begin{cases} -1, & \text{for } x \leq 0, \\ x - 1. & \text{for } x > 0. \end{cases}$$

(i) Draw the graph of $y = f(x)$. **2**

(ii) Find the value of $\int_{-2}^{2} f(x)dx$ **1**

(b) Following the birth of their daughter, Mr and Mrs Lee set up an account by investing \$40 000 which earns 8% interest per annum, compounded annually. They intend to withdraw \$$M$ every year starting from their daughter's 12th birthday until her 18th birthday. The first withdrawal will be on her 12th birthday and the last withdrawal will be on her 18th birthday.

(i) Find the amount of the money in the account just before the first withdrawal. **1**

(ii) Write an expression, in terms of M, for the amount of the money in the account immediately after the 2nd withdrawal. **1**

(iii) Write an expression, in terms of M, for the amount of the money in the account immediately after the 7th withdrawal on their daughter's 18th birthday. Simplify your answer. **3**

(iv) Calculate the value of M which leaves the account empty after the 7th withdrawal. **1**

(v) Suppose they wished to be able to withdraw at least \$23 400 every year instead of \$M and leave the account empty after the 7th withdrawal. **Estimate** the minimum interest rate per annum, correct to the nearest per cent, they would then need to earn on their account. **3**

End of paper

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STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \qquad n \neq -1; x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \qquad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \qquad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \qquad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \qquad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \qquad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \qquad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \qquad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \qquad a > 0, \qquad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \qquad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \qquad x > 0$

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